Eigensystem Synthesis for Active Flutter Suppression on an Oblique-Wing Aircraft

Gurbux S. Alag*
Western Michigan University, Kalamazoo, Michigan
and
John J. Burken†
NASA Ames Research Center, Dryden Flight Research Facility, Edwards, California

The application of the eigensystem synthesis technique to place the closed-loop eigenvalues and shape the closed-loop eigenvectors has not been practical for active flutter suppression, primarily because of the availability of only one control surface (aileron) for flutter suppression. The oblique-wing aircraft, because of its configuration, provides two independent surfaces (left and right ailerons), making the application of eigensystem synthesis practical. This paper presents the application of eigensystem synthesis using output feedback to design an active flutter suppression system for an oblique-wing aircraft. The results obtained are compared with those obtained by linear quadratic Gaussian techniques.

Nomenclature

A	= plant matrix
B	= control matrix
C	= output matrix
cgs	= center of gravity acceleration sensor
I	= identity matrix
K	= eigensystem gain matrix
L	= partitioned matrix of specified components of $(\lambda_i I - A)^{-1}B$
lws	= left wingtip acceleration sensor
$\bar{L}'$	= complex conjugate transpose of L
M	= partitioned matrix of unspecified components of $(\lambda_i I - A)^{-1}B$
q	= pitch
$\dot{q}$	= pitch rate
rws	= right wingtip acceleration sensor
u	= input vector
V	= eigenvector matrix
v_i	= closed-loop eigenvector
v_i^a, v_i^d	= achievable and desired eigenvectors respectively
v_i^*	= complex conjugate of v_i
x	= state vector
y	= output vector
λ_i, λ_i^*	= closed-loop eigenvalue and its conjugate
$\underline{\sigma}$	= minimum singular value of the return difference matrix

Received June 3, 1986; presented as Paper 86-2243 at the AIAA Guidance, Navigation, and Control Conference, Williamsburg, VA, Aug. 18-20, 1986; revision received Feb. 23, 1987.

*Associate Professor, Electrical Engineering Department. Member AIAA.
†Aerospace Engineer.

Introduction

THE U.S. Navy and NASA are currently involved in the design and development of an asymmetric skew-wing aircraft capable of a 65-deg wing sweep and flight at Mach 1.6 (see Fig. 1). Such a unique configuration exhibits aeroelastic behavior distinctly different from that of straight, swept-back, or swept-forward wings and has a potential for poor modal response characteristics. The most serious result of such characteristics can be flutter, an unstable motion caused by an interaction between structural vibrations and aerodynamic forces.[1] The active suppression of aerodynamic wing flutter can result in substantial weight savings and increases in performance compared with passive methods such as increased structural stiffness and mass balancing.

The synthesis of modal control systems for asymmetric aircraft requires considerably more states than are necessary for symmetric configurations. A symmetric configuration requires half the number of nodes to describe the equations of motion than does an asymmetric aircraft because the left and right sides are identical. The model representing the aircraft must include rigid-body modes, flexible modes, unsteady aerodynamics, actuators, and gust states. Control laws have previously been formulated for active flutter control using the standard linear quadratic Gaussian (LQG) procedure;[2-4] the resulting synthesized optimal feedback control laws are of the same order as the aircraft plant. The practical implementation of the control law requires order reduction by techniques such as transfer function matching, modal truncation, and residualization.[5-7]

To investigate modal control synthesis strategies for an oblique-wing configuration, a generic skew-wing aircraft model was developed for a 45-deg wing skew. The aircraft aerostructural model used in the process is a simple beam representation of the fuselage and wing. Shown in Fig. 2 are the node locations and the aeropanels superimposed over the beams. At a flight condition of Mach 0.70 and a 10,000-ft altitude, the model has an unstable flutter mode. An active, implementable flutter control law was developed[8] using the following methodology: 1) the formulation of the state-space model including independent wing actuators, a Dryden gust model, and s-plane approximations of unsteady aerody-

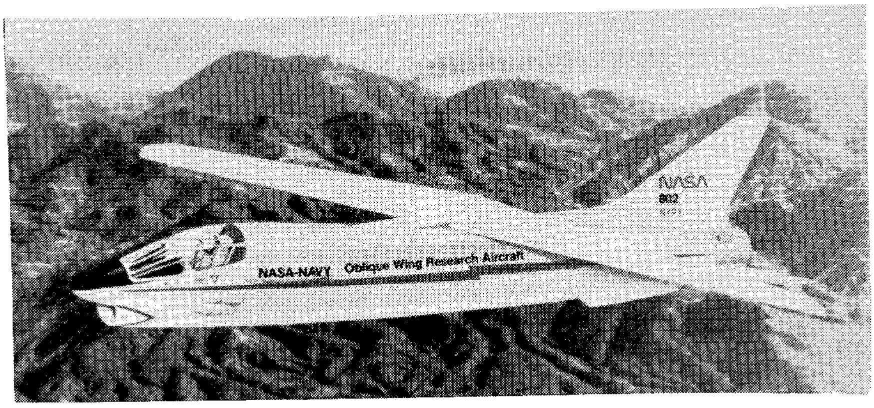

Fig. 1 Oblique wing aircraft.

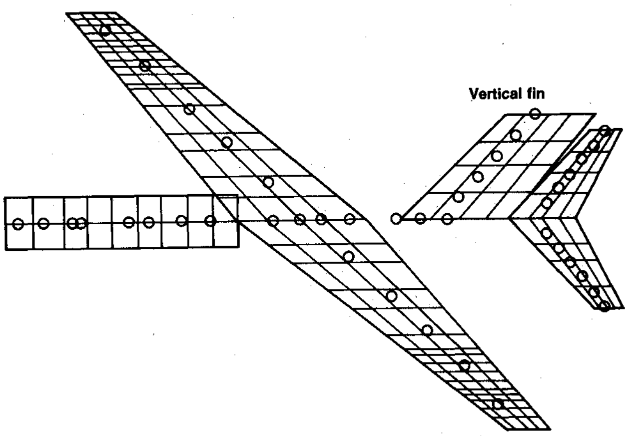

Fig. 2 Generic model (aeropanels and node points); vertical fin shown in x-y plane.

Table 1 Root-mean-square responses at flutter conditions

	Right wing		Left wing	
	δ, deg	$\dot{\delta}$, deg/s	δ, deg	$\dot{\delta}$, deg/s
Full-state feedback	1.58	7.91	0.28	3.58
Robust output controller	2.06	11.21	0.41	5.02
Reduced-order controller	1.51	10.04	0.41	4.90
Eigensystem synthesis	0.62	4.81	0.69	8.46

namics, 2) optimal full-state control law determination, 3) robust output feedback control law determination,[9] 4) reduced-order (practical) control law formulation, and 5) the evaluation of the practical control law.

The results of Ref. 8 are summarized in Fig. 3, which presents a comparison of the step responses of the robust full-order and reduced-order controllers. The degradation in the response due to controller size reduction can also be seen in the rms control surface activity. The rms responses of the controllers for a 1 ft/s gust input are shown in Table 1.

Eigensystem synthesis procedures are desirable for flight control system design because they do not increase the order of the system. Also, the difficulty of incorporating specifications such as damping, frequency, and decoupling within a quadratic performance index makes the eigensystem synthesis procedure a promising design alternative. The performance specifications can be interpreted in terms of desired closed-loop eigenvalues and eigenvectors. Moore[10] and others have shown how feedback can be used to place closed-loop eigenvalues and shape closed-loop eigenvectors. References 11-15 successfully demonstrate the use of an eigenstructure assignment procedure for aircraft control system design.

Liebst et al.[15] synthesized an active flutter control law using eigenvalue placement for a full-state controller. In addition, eigenvector shaping was used to enhance the gust load alleviation capabilities of the flutter control system.

The eigensystem synthesis technique using output feedback to place closed-loop eigenvalues and shape closed-loop eigenvectors has not been used for active flutter suppression, primarily because the availability of only one control surface for flutter suppression makes eigenvector shaping impractical. Output feedback eliminates the requirement of estimating the states. However, the oblique-wing aircraft, because of its asymmetric configuration, is ideally suited for the independent control of the left and right ailerons and makes the application of eigensystem synthesis practical.

This paper presents the application of eigensystem synthesis to the design of an active flutter suppression system for a generic model of an oblique-wing aircraft. The results obtained are compared with those obtained by LQG techniques.[8]

Flutter Suppression using Eigenstructure Techniques

The aircraft modal characteristics were developed using NASTRAN analysis. The unaugmented aircraft has an unstable flutter mode at a flight condition of Mach 0.7, 10,000 ft, and a wing skew of 45 deg, as shown in Fig. 2. The flutter mode is primarily wing bending. Because the intent of this paper is to demonstrate a design synthesis process, the model order was reduced considerably. The final model contained a rigid-body (primarily pitch) mode along with three elastic modes. The model reduction process did not significantly affect the flutter mode characteristics. Details of the aircraft model formulation are given in Ref. 8.

The aircraft design model includes the structural model, a linearized form of the unsteady aerodynamics, actuators, and gust model dynamics and can be represented as

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where x is a 24×1 state vector, y is a 5×1 output vector, u is a 2×1 input vector, and A, B, and C are the plant, control, and output matrices respectively of suitable dimensions. The 24 states include the rigid-body mode, flexible mode deflections, flexible mode rates, unsteady aerodynamic states, actuator deflection and rate states, and wind gust states. Eight states result from the retained structural modes, eight from the two-lag term set of approximated unsteady aerodynamics, six from the two actuators, and two from the gust

Table 2 Open-loop eigenvalues ($M=0.7$, $h=10,000$ ft)

0.0000 + 0.0000*i*
−0.4187 + 0.0000*i*
−0.4229 + 0.0000*i*
−4.2075 + 0.0000*i*
−0.1612 + 5.0036*i*
−0.1612 − 5.0036*i*
−1.9397 + 14.0551*i*
−1.9397 − 14.0551*i*
0.5011 + 14.3649*i*
0.5011 − 14.3649*i*
−20.0000 + 0.0000*i*
−20.0000 + 0.0000*i*
−30.8526 + 0.0000*i*
−35.1822 + 0.5223*i*
−35.1822 − 0.5223*i*
−37.1170 + 3.5647*i*
−37.1170 − 3.5647*i*
−39.6927 + 0.6080*i*
−39.6927 − 0.6080*i*
−41.3346 + 0.0000*i*
−36.4000 + 37.1354*i*
−36.4000 − 37.1354*i*
−36.4000 + 37.1354*i*
−36.4000 − 37.1354*i*

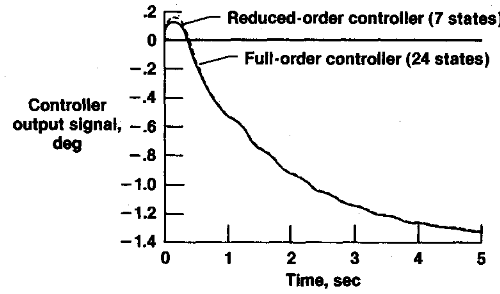

Fig. 3 Step response comparison of full-order and reduced-order controller.

model. The five outputs consist of the pitch angle, pitch rate, and three normal accelerations: the center of gravity and the right and left wingtips. The two inputs are right and left aileron deflections.

For the system under consideration, the following conditions hold:[12]

1) Five outputs permit the arbitrary assignment of up to five closed-loop eigenvalues with the stipulation that if λ_i is a complex closed-loop eigenvalue, its complex conjugate λ_i^* must also be a closed-loop eigenvalue.

2) A maximum of five eigenvectors can be altered. If the shape of a complex eigenvector v_i is altered, its complex conjugate v_i^* must be altered in the same way.

3) For each eigenvector whose shape is altered, a maximum of two eigenvector elements can be chosen arbitrarily because there are two controllers.

4) Achievable eigenvectors must lie in the two-dimensional subspace spanned by the columns of $(\lambda_i I-A)^{-1}B$. A desired eigenvector v_i^d will in general not reside in the achievable subspace. The optimal achievable eigenvector v_i^a is obtained by the orthogonal projection of v_i^d onto the subspace spanned by $(\lambda_i I-A)^{-1}B$.

Given the system of Eqs. (1) and (2) and assuming an output feedback, the control input u is given by

$$u=Ky \tag{3}$$

where K is the gain matrix of the dimension 2×5. For the closed-loop system, the following relationship holds:

$$(A-BKC)v_i=\lambda_i v_i \tag{4}$$

where v_i is a closed-loop eigenvector and λ_i a closed-loop eigenvalue, or

$$(\lambda_i I-A)v_i=-BKCv_i=BM_i \tag{5}$$

where

$$M_i=-KCv_i \tag{6}$$

In general, the desired eigenvector v_i^d does not reside in the achievable subspace. An approximate solution can be obtained by orthogonal projection.[13]

If

$$L_i=(\lambda_i I-A)^{-1}B \tag{7}$$

then the achievable eigenvector v_i^a is given by[13]

$$v_i^a=L_i(\bar{L}_i' L_i)^{-1}\bar{L}_i' v_i^d \tag{8}$$

and the gain K is given by

$$K=-M(CV)^{\dagger} \tag{9}$$

where the superscript † denotes pseudoinverse, $\bar{L}_i'$ is the complex conjugate transpose of L_i, and

$$M=[M_1\ M_2\ \ldots\ M_n] \tag{10}$$

$$V=[v_1^a v_2^a\ \ldots v_n^a] \tag{11}$$

Results

The outlined control law design process was used to synthesize an active flutter suppression controller for the oblique-wing aircraft model. The eigenvalues of the open-loop aircraft model are presented in Table 2. The unstable eigenvalue pair at this flight condition ($0.5\pm 14.37i$) represents primarily wing bending.

The design objective was to stabilize the aircraft without excessive root-mean-square (rms) control activity so that saturation would not occur. Based on practical actuator characteristics, the rms deflection of the aileron was limited to 5 deg and the deflection rate to 30 deg/s. In addition to stabilizing the aircraft with modest surface activity, it is required that the controller be robust. The controller considered here is multi-input, multi-output since the right and left wing control surfaces are independently controllable, the robustness of the multiloop control system is evaluated by using the singular values of the return difference matrix.[16]

The process of selecting a desirable set of closed-loop eigenvalues and eigenvectors for the case under consideration presents a challenge. The selection of desired eigenvalues and eigenvectors normally is based on engineering judgment and requires clear insight into the physical aspects of the plant being controlled. Significant physical insight for the flutter problem is lost in the process of model reduction from the original large dimensions to the 24th-order model developed for the control system synthesis process.

A time-consuming and arbitrary method for the selection of eigenvectors led to the stabilization of the aircraft. The achieved eigenvector was the projection of the desired eigenvector v_i^d onto the subspace spanned by the columns of $(\lambda_i I-A)^{-1}B$. The controller, however, was not very robust. The best method of selecting eigenvectors was by using the stable closed-loop eigenvalues and eigenvectors obtained through linear quadratic (LQ) design, assuming full-state feedback. The closed-loop eigenvalues and eigenvectors ob-

Table 3 Eigensystem variables

a) Desired eigenvectors (from LQ method)					b) Desired eigenvalues
3.9999	0.0438 + 0.0049i	0.0438 − 0.0049i	0.0015 + 0.0007i	0.0015 − 0.0007i	−0.0003 + 0.0000i
0.0000	−0.0317 + 0.0018i	−0.0317 − 0.0018i	−0.0245 + 0.0089i	−0.0245 − 0.0089i	−0.2355 − 5.0020i
0.0000	0.0011 + 0.0001i	0.0011 − 0.0001i	0.0023 + 0.0004i	0.0023 − 0.0004i	−0.2355 + 5.0020i
0.0000	0.0047 + 0.0001i	0.0047 − 0.0001i	−0.0004 + 0.0004i	−0.0004 − 0.0004i	−0.5041 − 14.3657i
−0.0010	−0.0349 + 0.2177i	−0.0349 − 0.2177i	−0.0104 + 0.0216i	−0.0104 − 0.0216i	−0.5041 + 14.3657i
0.0000	−0.0017 − 0.1588i	−0.0017 + 0.1588i	−0.1154 − 0.3565i	−0.1154 + 0.3565i	
0.0000	−0.0009 + 0.0057i	−0.0009 − 0.0057i	−0.0075 + 0.0330i	−0.0075 − 0.0330i	
0.0000	−0.0014 + 0.0236i	−0.0014 − 0.0236i	−0.0054 − 0.0065i	−0.0054 + 0.0065i	
0.0746	2.8445 + 16.2616i	2.8445 − 16.2616i	−6.5976 − 0.9621i	−6.5976 + 0.9621i	
0.0612	1.9139 + 5.3855i	1.9139 − 5.3855i	2.4618 + 6.5556i	2.4618 − 6.5556i	
−0.0247	−7.3179 − 74.9682i	−7.3179 + 74.9682i	2.4855 − 3.2472i	2.4855 + 3.2472i	
0.0829	0.9809 − 29.5832i	0.9809 + 29.5832i	10.8184 − 32.7421i	10.8184 + 32.7421i	
−0.0699	−2.6257 − 15.7088i	−2.6257 + 15.7088i	7.1162 + 3.8695i	7.1162 − 3.8695i	
−0.578	−1.6892 − 3.9659i	−1.6892 + 3.9659i	−2.0765 − 5.6525i	−2.0765 + 5.6525i	
0.0015	6.1769 + 73.0802i	6.1769 − 73.0802i	−2.6786 − 0.0123i	−2.6786 + 0.0123i	
−0.1024	−1.9195 + 26.9120i	−1.9195 − 26.9120i	−12.8369 + 31.1018i	−12.8369 − 31.1018i	
0.0000	0.0000 + 0.0001i	0.0000 − 0.0001i	0.0004 + 0.0003i	0.0004 − 0.0003i	
0.0000	−0.0003 + 0.0001i	−0.0003 − 0.0001i	−0.0050 + 0.0055i	−0.0050 − 0.0055i	
0.0000	−0.0006 − 0.0016i	−0.0006 + 0.0016i	−0.0766 − 0.0740i	−0.0766 + 0.0740i	
0.0000	0.0000 + 0.0000i	0.0000 − 0.0000i	0.0009 − 0.0001i	0.0009 + 0.0001i	
0.0000	0.0000 + 0.0000i	0.0000 − 0.0000i	0.0016 + 0.0130i	0.0016 − 0.0130i	
0.0000	−0.0001 − 0.0001i	−0.0001 + 0.0001i	−0.1878 + 0.0159i	−0.1878 − 0.0159i	
0.0000	0.0000 + 0.0000i	0.0000 − 0.0000i	0.0000 − 0.0000i	0.0000 + 0.0000i	
0.0000	0.0000 + 0.0000i	0.0000 − 0.0000i	0.0000 − 0.0000i	0.0000 + 0.0000i	

c) Eigensystem feedback gain matrix, K

cgs	lws	rws	q	$\dot{q}$
0.00003	0.00041	−0.00057	−0.00009	0.00015
0.00003	0.00090	−0.00103	−0.00013	−0.00014

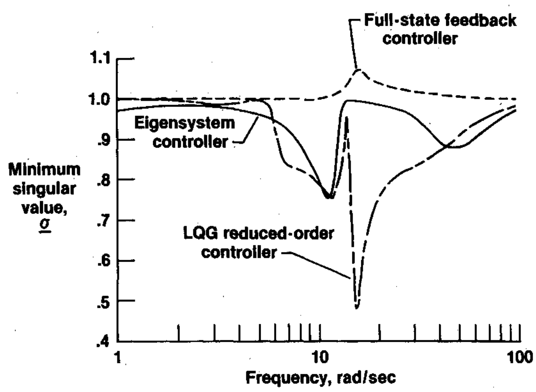

Fig. 4 Minimum singular values of the return difference matrix.

tained by this method were selected as the desired eigenvalues and eigenvectors for the eigensystem synthesis process. Since the eigenvalues and eigenvectors desired are a subset of the closed-loop eigenvalues and eigenvectors obtained by LQ solution, these eigenvectors are achievable and no orthogonal projection is required in this case.

Tables 3a and 3b give the desired locations of the five closed-loop eigenvectors as well as the eigenvalues. The desired eigenvectors and eigenvalues were achieved with the gain matrix K given in Table 3c.

Table 1 presents a comparison description of the rms values of the surface response to a gust input for both the LQG reduced-order design[8] (seventh-order controller) and the controller designed using eigensystem synthesis. The eigensystem synthesis rms values for the surface deflections and rates are within the specified range and compare favorably with those obtained by the LQG method.

The eigensystem controller (K is a 2×5 matrix) is simple to implement and does not increase the order of the system. In comparison, the LQG design technique uses a full-order controller (the order being the same as that of the plant). Even a reduced-order controller is difficult to implement and increases the order of the system.

However, this eigensystem synthesis approach comprises robustness. The stability robustness of multi-input, multi-output feedback control systems is characterized by the minimum singular value of the return difference matrix of the plant input or output.[16] Figure 4 shows the plots of the minimum singular values $\underline{\sigma}$ for the controller developed in this paper and for the full-state feedback controller developed using the LQG technique.[8] A degradation in robustness is evident from the plot. However, the LQG reduced-order controller is not as robust as the one designed using the eigensystem procedure, as is evident from Fig. 4. The stability margins of the LQG reduced-order controller are acceptable.

Conclusions

An implementable flutter controller for a 45-deg skew oblique-wing aircraft mathematical model was designed using the eigensystem synthesis technique. The controller does not increase the order of the system and is simple to implement. Future studies include improving the stability margins by using constrained optimization techniques to shape the singular-value spectrum.[17] A standard performance index is minimized while trying to satisfy minimum singular value constraints at the plant input and/or output.

References

[1]Edwards, J.W., Breakwell, J.V., and Bryson, A.E., "Active Flutter Control Using Generalized Unsteady Aerodynamic Theory," *Journal of Guidance and Control*, Vol. 1, Jan.-Feb. 1978, pp. 32-40.

[2]Adams, W.M. Jr. and Tiffany, S.H., "Design of a Candidate Flutter Suppression Control Law for DAST ARW-2," AIAA Paper 83-2221, Aug. 1983.

[3]Mahesh, J.K., Stone, C.R., Garrard, W.L., and Dunn, H.J., "Control Law Synthesis for Flutter Suppression Using Linear Quadratic Gaussian Theory," *Journal of Guidance Control,* Vol. 4, 1981, pp. 415-422.

[4]Mukhopadhyay, V., Newson, J.R., and Abel, I., "Reduced-Order Optimal Feedback Control Law Synthesis for Flutter Suppression," *Journal of Guidance Control,* Vol. 5, 1982, pp. 389-395.

[5]Newsom, J.R., "A Method for Obtaining Practical Flutter Suppression Control Laws Using Results of Optimal Control Theory," NASA TP-1471, 1979.

[6]Newsom, J.R., Adams, W.M. Jr., Mukhopadhyay, V., Tiffany, S.H., and Abel, I., "Active Controls: A Look at Analytical Methods and Associated Tools," NASA TM-86269, 1984.

[7]Mahesh, J.K., Stone, C.R., Garrard, W.L., and Hausman, P.D., "Active Flutter Control for Flexible Vehicles," NASA CR-159160, 1979.

[8]Burken, J.J., Alag, G.S., and Gilyard, G.B., "Aeroelastic Control of Oblique-Wing Aircraft," *Proceedings of the American Control Conference,* 1986, Seattle, WA, pp. 463-471.

[9]Doyle, J.C. and Stein, G., "Robustness with Observers," *IEEE Transactions on Automatic Control,* Vol. AC-23, Aug. 1979, pp. 756-757.

[10]Moore, B.C., "On the Flexibility Offered by Full State Feedback in Multivariable Systems Beyond Closed Loop Eigenvalue Assignment," *IEEE Transactions on Automatic Control,* Vol. 21, Oct. 1976, pp. 689-692.

[11]Cunningham, T.B., "Eigenspace Selection Procedures for Closed Loop Response Shaping With Modal Control," *Proceedings of the 19th IEEE Conference on Decision and Control,* Dec. 1980, pp. 178-186.

[12]Andry, A.N., Shapiro, E.Y., and Chung, J.C., "Eigenstructure Assignment for Linear Systems," *IEEE Transactions on Aerospace Electronic Systems,* Sept. 1983, pp. 711-729.

[13]Sobel, K.M. and Shapiro, E.Y., "Eigenstructure Assignment for Design of Multimode Flight Control Systems," *IEEE Control Systems Magazine,* Vol. 5, May 1985.

[14]Harvey, C.A., Stein, G., and Doyle, J.C., "Optimal Linear Control," Report ONR CR 215-238-2, 1977.

[15]Liebst, B.M., Garrard, W.L., and Adams, W.M., "Design of an Active Flutter Suppression System," *Journal of Guidance Control,* Vol. 9, Jan.-Feb. 1986.

[16]Ly, U.-L., "Robustness Analysis of a Multiloop Flight Control System," AIAA-83-2189, Aug. 1983.

[17]Mukhopadhyay, V., "Stability Robustness Improvement Using Constrained Optimization Techniques," AIAA-85-1931-CP, Aug. 1985.